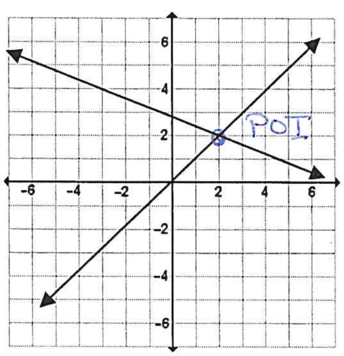
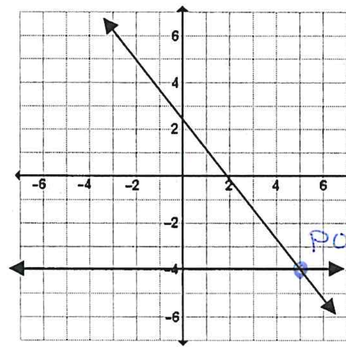
INTRODUCTION TO LINEAR SYSTEMS

Some problems are solved by graphing linear equations on the Cartesian Plane and finding where they cross (i.e., finding the point of intersection).
The **point of intersection** is called the **solution of a linear system of equations.**

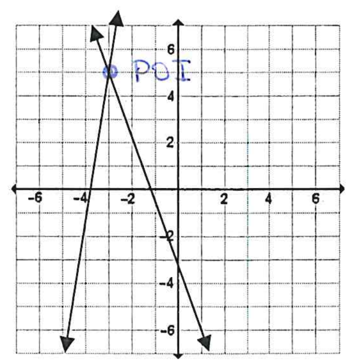
State the solution of each of the following linear systems:



(2, 2)



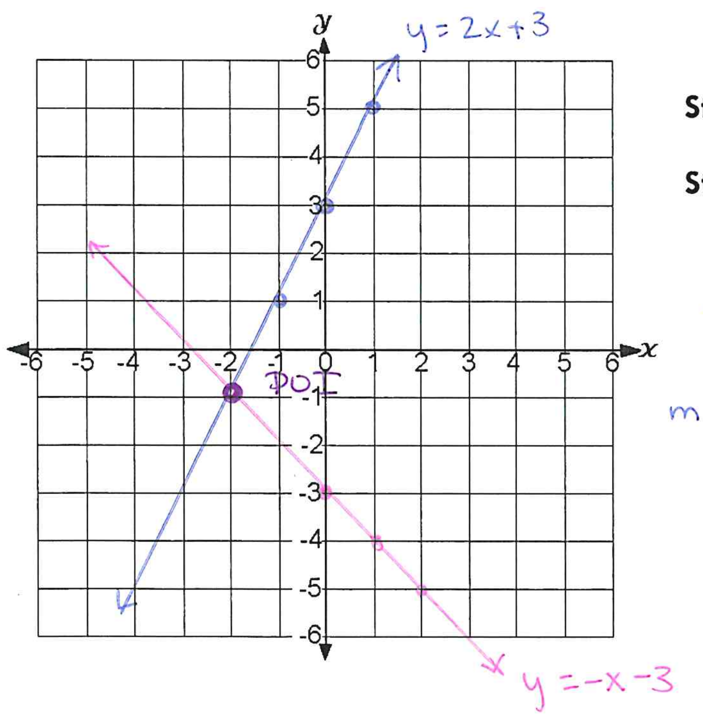
(5, −4)



(−3, 5)

EXAMPLE: Solve the following linear system:

Both the Girl Guides and Boy Scouts are hiking through Algonquin Park.
The Girl Guides are travelling in a direction that is represented by the equation **$y = 2x + 3$.**
The Boy Scouts are travelling in a direction that is represented by the equation **$y = -x - 3$.**
Find the coordinates of the point where their paths will cross.



Step 1: Plot both equations on the graph.

Step 2: Find the point of intersection.

$y = 2x + 3$ $\quad$ $m = \frac{2}{1}$ $\quad$ $b = 3$

$y = -x - 3$ $\quad$ $m = \frac{-1}{1}$ $\quad$ $b = -3$

POI = (−2, −1)

∴ Their paths will cross at (−2, −1)

SOLVING LINEAR SYSTEMS BY GRAPHING

A **linear system** is represented by at least two linear equations (lines)

The **Point of Intersection** (POI) of the two lines is the Solution of a linear system of equations.

The **POI** Point of Intersection will make each equation TRUE when the (x, y) values are substituted into each equation.

You can **CHECK** to see if your solution is correct by Substituting the solution back into your equations to see if the LS of the equation equals the RS.

LS	RS

There are several ways to find the solution to a linear system of equations.

1) By Graphing
2) By Substitution
3) By Climination

Let's begin with GRAPHING

Solve each linear system graphically.
Remember to rearrange the equations, if necessary, into the y = mx + b form first.

Example 1

$x + y = 6$ and $2x - y = 0$

$y = -x + 6$

$m = \frac{-1}{1}$

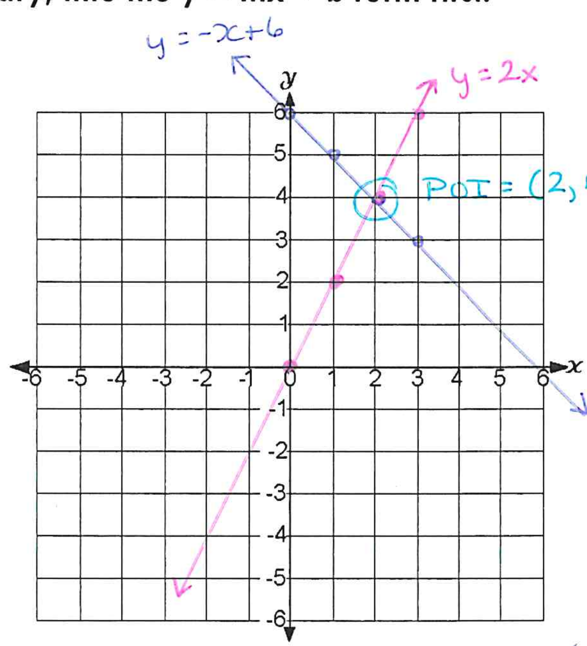
$b = 6$

$\frac{-y}{-1} = \frac{-2x}{-1} + \frac{0}{-1}$

$y = 2x + 0$

$m = \frac{2}{1}$

$b = 0$



$\therefore$ The POI is $(2, 4)$

Check: $(2, 4)$ — x, y

LS	RS
$= x + y$	6
$= (2) + (4)$	
$= 6$	✓

LS = RS

LS	RS
$= 2x - y$	$= 0$
$= 2(2) - (4)$	
$= 4 - 4$	✓
$= 0$	

LS = RS

Example 2

$y = x + 3$ and $3x + y + 1 = 0$

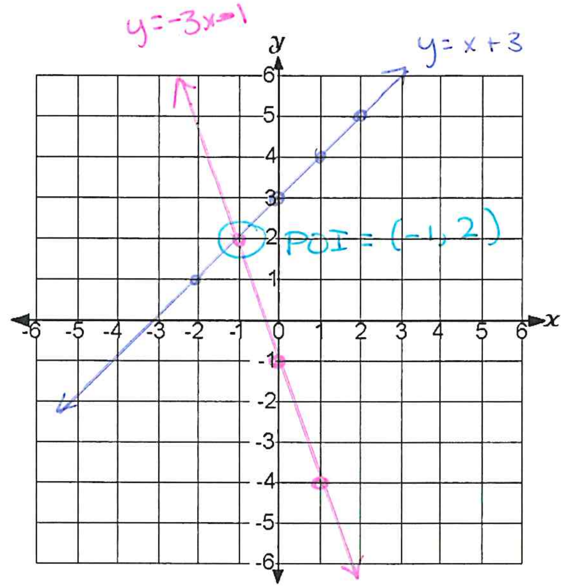
$m = \frac{1}{1}$

$b = 3$

$y = -3x - 1$

$m = \frac{-3}{1}$

$b = -1$



∴ The POI is $(-1, 2)$

Check: $(\overset{x}{-1}, \overset{y}{2})$

LS	RS
$= y$	$= x + 3$
$= (2)$	$= (-1) + 3$
$= 2$	$= 2$

LS = RS ✓

LS	RS
$= 3x + y + 1$	0
$= 3(-1) + (2) + 1$	
$= -3 + 2 + 1$	
$= 0$	

LS = RS

Example 3

$2y - 3x = 12$ and $-x + 4y = 4$

$\frac{2y}{2} = \frac{3x}{2} + \frac{12}{2}$

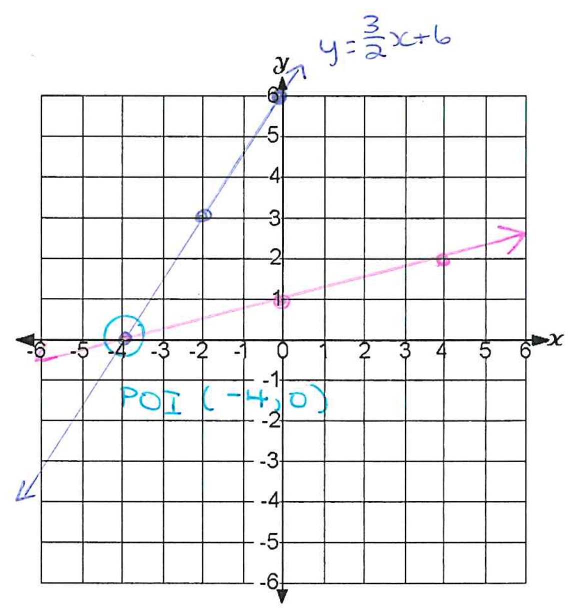
$y = \frac{3}{2}x + 6$

$m = \frac{3}{2}$ $b = 6$

$\frac{4y}{4} = \frac{x}{4} + \frac{4}{4}$

$y = \frac{1}{4}x + 1$

$m = \frac{1}{4}$ $b = 1$



∴ The POI is $(-4, 0)$

Check: $(\overset{x}{-4}, \overset{y}{0})$

LS	RS
$2y - 3x$	12
$= 2(0) - 3(-4)$	
$= 0 + 12$	
$= 12$	

LS = RS

LS	RS
$-x + 4y$	4
$= -(-4) + 4(0)$	
$= 4 + 0$	
$= 4$	

LS = RS